E023 NONLINEAR WAVEFORM INVERSION VIA MODIFIED NEWTON METHOD BASED ON TRUNCATED SVD

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The modified Newton method to solve the nonlinear equation $A\ (c) = u_0$ consists of an iterative process aimed at getting successive approximation by resolving the linear equation.

$$DA\ (c_0) < c_k+1 - c_k = A\ (c_k) - u_0 \qquad (1)$$

with $DA\ (c_0)$ being the derivative of A at the starting point c_0. However its straightforward application in order to resolve inverse problems of wave propagation is senseless as DA has no bounded inverse. Thus its numerical implementation should stipulate some regularization procedure. For this, we use the r-pseudoinverse - the generalized normal inverse of operator $(DA\ (c_0)\ r)$ that can be obtained by means of the truncated SVD of the operator $DA\ (c_0)$ (r is the rank of $(DA\ (c_0))r)$ ([1]). With this regularization procedure, the iterative process for the modified Newton method is represented as follows:

$$c_k+1 = c_k - (DA\ (c_0)\ r^{\dagger} < u_0 - a\ (c_k) >. \qquad (2)$$

Under some assumptions ([2]), the sequence c_k becomes $P_r < c_* >$, where c_* is the exact solution of the inverse problem (wave propagation velocity) and P_r is an orthogonal projector onto the linear span of r right singular vectors of the derivative $DA\ (c_0)$. The rank r is determined by the noise level of the input data and by the accuracy of the finite dimensional approximation of the operator A and its derivative DA. It should be noted that the first step of the iterative process (2) is a linearized waveform inversion (Born inversion) with an inhomogeneous background treated by a range of authors (see, for example [3]).

This approach was implemented and tested numerically for 2D nonlinear waveform inversion of synthetic surface multicoverage data simulated by means of finite differences on a parallel computer Parsytec PowerXplorer (8 nodes with PowerPC-601, 80Mh, 8Mb memory of each node). The real velocity model was represented as a local object (ellipse with axes of 6 and 3 wavelengths) embedded in a vertically inhomogeneous background. The velocity deviation within this object was taken as +25% in comparison with the background. The value of the parameter r was chosen so that the condition number of $DA\ (c0)^{\dagger}_r$ was no more than 1000.

Fig.1 represents the result of the first step of process (2), while Fig.2 is the ω - k prestack migration. As one can see, the first image depicts not only the boundary of the object, but also its interior structure. Of course, one can see some low frequency oscillations in the reconstructed model of the wave propagation velocity because of the lack of low time frequencies in the source function (Ricker impulse with dominant frequency 30Hz), the displacement of the lower boundary of the object and blurring of its edges. Successive iterations (2) move the lower boundary to its correct position and diminish the blurring of the edges, but do not improve essentially the low spatial frequency components.

In order to analyse the capability of this approach to reconstruct specific spatial frequencies of the model, we computed the mutual disposition of the linear span of the right singular vectors of the operator $DA\ (c_0)$ for a chosen value of the parameter r with respect to a standard Fourier basis in lateral and vertical coordinates. Since not only are the high spatial vertical harmonics almost orthogonal to this linear span (Rayleigh's criteria), but also the lower ones (problem of trend reconstruction), there is no hope of reconstructing the smooth vertical components of the model by means of successive iterations (2).

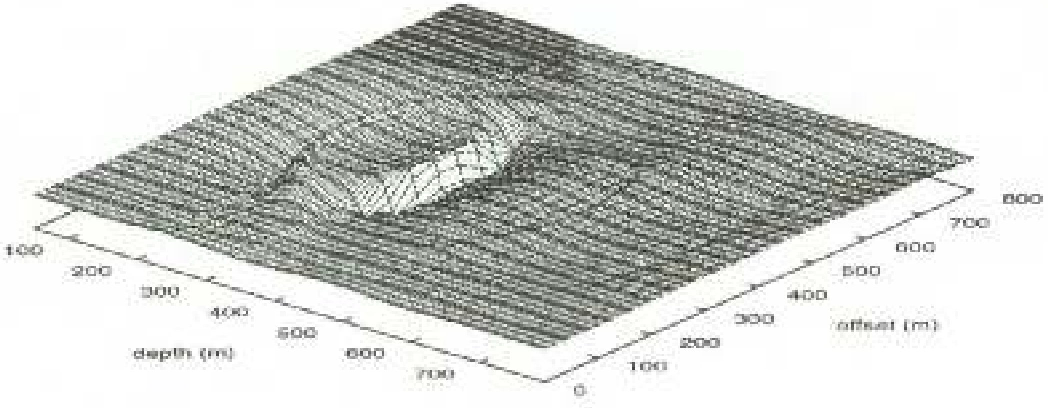

Figure 1: Result of first iteration

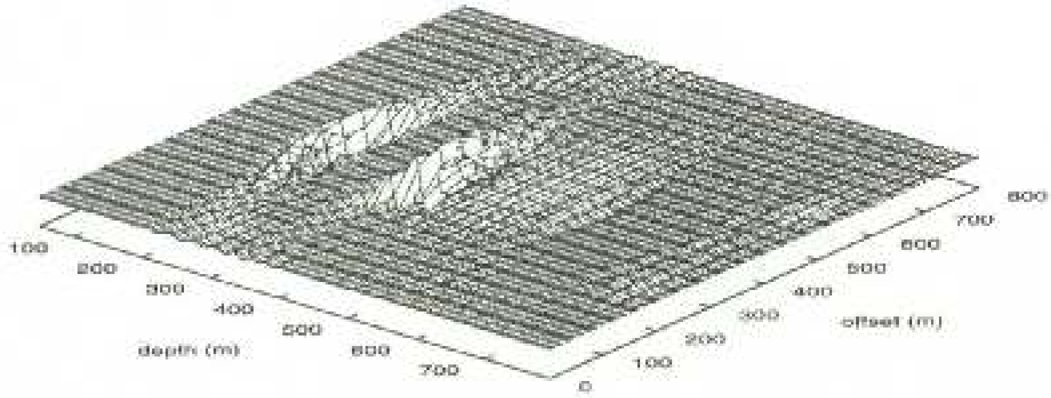

Figure 2: $\omega - k$ prestack migration

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References

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